DATE : / /
PAGE :

For unity feedback system.

$$H(s) = 1$$

Then

$$\boxed{e_{ss} = \lim_{s \to 0} s \frac{R(s)}{1 + G(s)}}$$

Thus, from above equation it is clear that the steady state error depends upon input and closed loop transfer function.

STATIC ERROR COEFFICIENTS

Static position error constant (or coefficient) K_p.

∴ we know that,

$$e_{ss} = \lim_{s \to 0} s \cdot \frac{R(s)}{1 + G(s) H(s)}.$$

For unit step input

$$R(s) = \frac{1}{s}$$

$$e_{ss} = \lim_{s \to 0} \cancel{s} \cdot \frac{1}{\cancel{s}(1 + G(s) \cdot H(s))}$$

$$e_{ss} = \frac{1}{1 + \lim_{s \to 0} G(s) \cdot H(s)}$$

put $K_p = \lim_{s \to 0} G(s) \cdot H(s)$ is called static position error constant.

∴ $e_{ss} = \lim_{s \to 0}$

$$e_{ss} = \frac{1}{1 + K_p}$$

and

$$K_p = \lim_{s \to 0} G(s) \cdot H(s)$$

II) Static velocity error constant (or coefficient) K_v

∴ we know that
The ~~static~~ steady state error is given by:

$$e_{ss} = \lim_{s \to 0} s \cdot \frac{R(s)}{1 + G(s) \cdot H(s)}$$

for unit ramp input

$$R(s) = \frac{1}{s^2}$$

put in above equation we get:

$$e_{ss} = \lim_{s \to 0} s \cdot \frac{1}{s^2[1 + G(s) \cdot H(s)]}$$

$$= \lim_{s \to 0} \frac{1}{s[1 + G(s) \cdot H(s)]}$$

$$= \lim_{s \to 0} \frac{1}{s + s G(s) \cdot H(s)} = \lim_{s \to 0} \frac{1}{[0 + s G(s) \cdot H(s)]}$$

$$= \lim_{s \to 0} \frac{1}{G(s) H(s) \cdot s}$$

$$= \frac{1}{K_v}$$ where $K_v = \lim_{s \to 0} \frac{1}{s \cdot G(s) \cdot H(s)}$

Hence $\boxed{K_v = \frac{1}{\lim_{s \to 0} s G(s) \cdot H(s)}}$ is called static velocity constant

and $\boxed{e_{ss} = \frac{1}{K_v}}$

Static acceleration error constants (coefficient): K_a

The steady state error of system with unit parabolic input is given by:

$$e_{ss} = \lim_{s \to 0} s \cdot \frac{R(s)}{1 + G(s) \cdot H(s)}$$

For unit parabolic input

$$R(s) = \frac{1}{s^2}$$

$$\therefore e_{ss} = \lim_{s \to 0} s \cdot \frac{1}{s^2 \cdot [1 + G(s) \cdot H(s)]}$$

$$= \lim_{s \to 0} \frac{1}{s^2 (1 + G(s) \cdot H(s)]}$$

$$= \frac{1}{0 + [\lim_{s \to 0} s^2 G(s) \cdot H(s)]}$$

put $\lim_{s \to 0} s^2 G(s) \cdot H(s) = K_a$

$$\boxed{e_{ss} = \frac{1}{K_a}}$$

Static acceleration constant

$$\boxed{K_a = \lim_{s \to 0} s^2 G(s) \cdot H(s)}$$

CLASSIFICATION OF CONTROL SYSTEM

Consider the open loop transfer function

$$G(s) H(s) = \frac{K(1+sT_1)(1+sT_2) \ldots}{s^m (1+sT_a)(1+sT_b) + \ldots} \quad \text{—— (1)}$$

The poles of the system are at $s = \frac{-1}{T_a}, \frac{-1}{T_b}$ and zeros at $s = -\frac{1}{T_1}, \frac{1}{T_2}$

The term 's^m' gives the number of ~~So~~ poles at origin.

Type '0' system:- A system having no pole at origin of the 's' plane is said type '0' system. i.e $m=0$

Type '1' system:- If a system having a single pole at origin of 's'-plane is said to be type '1' system.
For type-1 system $s=1$.

Type '2' systems:- ~~As~~ If a system having two poles at origin of the s-plane is called type-2 system.
For type-2 system, $m=2$.

1) Type '0' system with unit step input:-
∴ Open loop transfer function is given by

$$G(s)H(s) = \frac{K(1+sT_1)(1+sT_2)\ldots}{(1+sT_a)(1+sT_b)\ldots}$$

$$\therefore K_p = \lim_{s\to 0} G(s)\cdot H(s)$$

$$= \lim_{s\to 0} \frac{K(1+sT_1)(1+sT_2)\ldots}{(1+sT_a)(1+sT_b)+\ldots} = K.$$

$$\therefore e_{ss} = \frac{1}{1+K_p} = \frac{1}{1+K}$$

$$\boxed{e_{ss} = \frac{1}{1+K}}$$

2) Type '0' system with unit ramp input:

∴ As we know that:

$$K_v = \lim_{s \to 0} s\, G(s) H(s)$$

$$= \lim_{s \to 0} s \cdot \frac{K(1+sT_1)(1+sT_2)\cdots}{(1+sT_a)(1+sT_b)\cdots} = 0$$

$$\therefore \quad e_{ss} = \frac{1}{K_v} = \frac{1}{0} = \infty$$

Type '0' system with unit parabolic input:

$$\therefore \quad K_a = \lim_{s \to 0} s^2 G(s) H(s)$$

$$= \lim_{s \to 0} s^2 \frac{K(1+sT_1)(1+sT_2)\cdots}{(1+sT_a)(1+sT_b)\cdots}$$

$$= 0$$

$$\therefore \quad e_{ss} = \frac{1}{K_a} = \frac{1}{0} = \infty$$

Type '1' system ~~with~~ unit

Type '1' system with unit step input

$$\therefore \quad G(s)H(s) = \frac{K(1+sT_1)(1+sT_2)}{s(1+sT_a)(1+sT_b)}$$

∴ we know that:

$$K_p = \lim_{s \to 0} G(s) \cdot H(s)$$

$$= \lim_{s \to 0} \frac{K(1+sT_1)(1+sT_2)\cdots}{s(1+sT_a)(1+sT_b)\cdots}$$

$$= \infty$$

$$\therefore \quad e_{ss} = \frac{1}{1+K_p} = \frac{1}{1+\infty} = 0$$

2) Type-1 system with unit ramp input:

$$\therefore \quad K_v = \lim_{s \to 0} s\,G(s)\,H(s)$$

$$= \lim_{s \to 0} \frac{\cancel{s} \cdot k(1+sT_1)(1+sT_2)\ldots}{\cancel{s}(1+sT_a)(1+sT_b)}$$

$$= k.$$

$$\therefore \quad e_{ss} = \frac{1}{K_v} = \frac{1}{k}.$$

3) Type-1 system with unit parabolic input:

$$\therefore \quad K_a = \lim_{s \to 0} s^2\,G(s)\cdot H(s)$$

$$= \lim_{s \to 0} \frac{\cancel{s^2}\, k(1+sT_1)(1+sT_2)\ldots}{\cancel{s}(1+sT_a)(1+sT_b)\ldots}$$

$$= \lim_{s \to 0} \frac{sk(1+sT_1)(1+sT_2)\ldots}{(1+sT_a)(1+sT_b)\ldots}$$

$$= 0$$

$$\therefore \quad e_{ss} = \frac{1}{k_a} = \frac{1}{0} = \infty$$

Type '2' system

1) With unit step input.

$\therefore$ we know that for type-2 system

$$G(s)\,H(s) = \frac{k(1+sT_1)(1+sT_2)\ldots}{s^2(1+sT_a)(1+sT_b)\ldots}$$

$$\therefore \quad k_p = \lim_{s \to 0} G(s)\cdot H(s) = \infty$$

$$\therefore \quad e_{ss} = \frac{1}{1+k_p} = \frac{1}{1+\infty} = 0$$

With unit ramp input:

$$\therefore K_v = \lim_{s\to 0} s\,G(s)H(s)$$

$$= \lim_{s\to 0} \frac{\cancel{s}\,K(1+sT_1)(1+sT_2)\cdots}{s^{\cancel{2}}(1+sT_a)(1+sT_b)\cdots}$$

$$= \lim_{s\to 0} \frac{1}{s}\,\frac{K(1+sT_1)(1+sT_2)\cdots}{(1+sT_a)(1+sT_b)\cdots}$$

$$= \infty$$

$$\therefore e_{ss} = \frac{1}{K_v} = \frac{1}{\infty} = 0$$

3) With unit ~~ramp~~ parabolic input

$$\therefore K_a = \lim_{s\to 0} s^2\,G(s)H(s)$$

$$= \lim_{s\to 0} \frac{\cancel{s^2}\,K(1+sT_1)(1+sT_2)\cdots}{\cancel{s^2}(1+sT_a)(1+sT_b)\cdots}$$

$$= K$$

$$\therefore e_{ss} = \frac{1}{K_a} = \frac{1}{K}$$

	Type '0' system	Type '1' system	Type '2'
Unit step input	$\frac{1}{1+K}$	0	0
Unit ramp input	∞	$\frac{1}{K}$	0
Unit parabolic input	∞	∞	$\frac{1}{K}$